ALGEBRAIC TOPOLOGY I WS23/24, HOMEWORK SHEET 10

DEADLINE: FRIDAY, DECEMBER 22ND

Problem 1.

(1) As defined in the lecture, let $\gamma_{\mathbb{R}}^{1,n+1}$ denote the tautological bundle over $\mathbb{R}P^n$. Prove that the Thom space $Th(\gamma_{\mathbb{R}}^{1,n+1})$ is homeomorphic to $\mathbb{R}P^{n+1}$. Show this also in the limit case $n = \infty$, i.e., show that the Thom space of the universal line bundle $\gamma_{\mathbb{R}}^{1}$ is again homeomorphic to $\mathbb{R}P^{\infty}$.

(2) Use the Thom isomorphism and Part (1) to give an alternative proof that the ring $H^*(\mathbb{R}P^{\infty}, \mathbb{F}_2)$ is polynomial on a class in degree 1.

Problem 2. Let ξ be a vector bundle over a compact base space B (recall that for us 'compact' in particular means that B is a Hausdorff space).

(1) Show that the total space $E = E(\xi)$ is locally compact and Hausdorff, and hence its one-point compactification E^+ is defined.

(2) Prove that the Thom space $Th(\xi)$ is homeomorphic to E^+.